Code No: 183BZ **R22**

JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY HYDERABAD
B. Tech II Year I Semester Examinations, February - 2024
PROBABILITY, STATISTICS AND COMPLEX VARIABLES
(Common to ME, AE, MIE)

Time: 3 Hours **Max. Marks: 60**

Note: This question paper contains two parts A and B.
i) **Part- A** for 10 marks, ii) **Part - B** for 50 marks.
- Part-A is a compulsory question which consists of ten sub-questions from all units carrying equal marks.
- Part-B consists of **ten questions** (numbered from 2 to 11) **carrying 10 marks each**. From each unit, there are two questions and the student should answer one of them. Hence, the student should answer five questions from Part-B.

PART- A

(10 Marks)

1.a)	Define conditional probability with one example.	[1]
b)	Define discrete and continuous random variables.	[1]
c)	Write the mean, median and mode of a normal distribution.	[1]
d)	Write the mean and variance of a Poisson distribution.	[1]
e)	Define null and alternate hypotheses.	[1]
f)	A hypothesis is false but accepted. What type of error it is?	[1]
g)	Write Cauchy-Riemann equations.	[1]
h)	Describe conformal mapping.	[1]
i)	State Cauchy Residue theorem.	[1]
j)	Write Cauchy's Integral formula.	[1]

PART - B

(50 Marks)

2.a) A and B throw alternately a pair of dice. A wins if he throws 6 before B throws 7 and B wins if he throws 7 before A throws 6. If A begins, find his chance of winning.

b) A random variable X has the following probability function:

Values of x :	- 2	-1	0	1	2	3
p(x) :	0.1	k	0.2	2k	0.3	k

Find the value of k, and calculate mean and variance. [5+5]

OR

3.a) In a bolt factory, there are four machines A, B, C, D manufacturing 20%, 15%, 25% and 40% of the total output respectively. Of their outputs 5%, 4%, 3% and 2%′ in the same order are defective bolts. A bolt is chosen at random from the factory's production and is found defective. What is the probability that the bolt was manufactured by machine A or machine D?

b) X is a continuous random variable with probability density function given by

$$f(x)=\begin{cases} kx & (0 \le x < 2) \\ 2k & (2 \le x < 4) \\ -kx + 6k & (4 \le x < 6) \end{cases}$$

Find k and mean value of X. [5+5]

4.a) Find the mean and variance of exponential distribution.

b) For a normally distributed variate with mean 1 and S.D. 3, find the probabilities that (i) $3.43 \leq x \leq 6.19$ (ii) $-1.43 \leq x \leq 6.19$. [5+5]

OR

5.a) The incidence of occupational disease in an industry is such that the workmen have a 10% chance of suffering from it. What is the probability that in a group of 7, five or more will suffer from it?

b) In a normal distribution, 31% of the items are under 45 and 8% are over 64. Find the mean and S.D. of the distribution. [5+5]

6.a) The nine items of a sample have the following values: 45, 47, 50, 52, 48, 47, 49, 53, 51. Does the mean of these differ significantly from the assumed mean of 47.5?

b) The mean yield of wheat from a district A was 210 pounds with SD 10 pounds per acre from a sample of 100 plots. In another district B, The mean yield was 200 pounds with SD 12 pounds from a sample of 150 plots. Assuming that the SD of yield in the entire state was 11 pounds. Test whether there is any significant difference between the mean yield of crops in the two districts. [5+5]

OR

7.a) The IQs (intelligence quotients) of 16 students from one area of a city showed a mean of 107 with SD of 10 , while the IQs of 14 students from another area of the city showed a mean of 112 with a SD of 8. Is there a significant difference between the IQs of the two groups at a 5% LoS?

b) Random samples of 400 men and 600 women were asked whether they would like to have a flyover near their residence. 200 men and 325 women were in favour of the proposal. Test the hypothesis that proportions of men and women in favour of the proposal are same, at 5% level. [5+5]

8.a) Find the analytic function, whose real part is $\frac{\sin 2x}{(\cosh 2y - \cos 2x).}$

b) Find the Mobius transformation which maps the points z= l, i, -1 onto the points w = i, 0, - i. Hence find the invariant points of this transformation. [5+5]

OR

9.a) Show that the function $u = e^{-2xy} \sin(x^2 - y^2)$ is harmonic. Find the conjugate function v and express u + iv as an analytic function of z.

b) If w = log z, find dw/dz and determine where w is non-analytic. [5+5]

10.a) Find Taylor's expansion of $f(z) = \frac{2z^3+1}{z^2+z}$ about the point z=i.

b) Evaluate $\oint_C \frac{4z^2 - 4z + 1}{(z-2)(4+z^2)} dz$, where C is | z | = 1. [5+5]

OR

11.a) Find the Laurent's series expansion of $f(z) = \frac{z^2-6z-1}{(z-1)(z-3)(z+2)}$ in the region $3 < |z+2| < 5$.

b) Find the sum of the residues of $f(z) = \frac{\sin z}{z \cos z}$ at its poles inside the circle |z| =2. [5+5]

---ooOoo---